

Gauss-Bessel Beam Shaper [Power Control]

Digital Twin Specification

Twin Code:	CF-BESP01
Twin Name:	Gauss-Bessel Beam Shaper [Power Control]
Category:	Component
Type:	Function-Based
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Description

This twin implements a quantized radial phase mask that converts a collimated Gaussian beam into a Bessel-like beam with extended depth of focus. In a $2f$ configuration, the mask placed at the front focal plane produces the exact Fourier transform of the shaped field. The ideal Bessel beam requires a linear radial phase $\phi_{\text{ideal}}(\rho) = k_{\rho}\rho$; here the phase is quantized into Q levels. Lower Q (e.g., binary) yields multiple diffraction orders, while higher Q approaches a clean Bessel ring with a J_0^2 profile.

A key feature is the phase scaling factor α (modulation depth):

- $\alpha = 0$: pure Gaussian (all energy in central spot).
- $0 < \alpha < 1$: energy split between spot and rings.
- $\alpha = 1$: pure Bessel generator (energy ideally in rings).

Thus α provides simple control over the energy distribution. The output polarization is unchanged.

Model Parameters

Design parameters:

- **Ring separation ratio (M):** Controls the spacing between rings relative to the diffraction-limited focal spot size (default: 2).
 - $M = 1$: First ring just touches central spot (minimum resolution)
 - $M = 2$: Clear separation between rings (recommended default)
 - $M = 3$: Multiple rings well resolved
 - Warning issued if $M < 1$ (rings not resolved)
- **Phase scaling factor (α):** Modulation depth (default: 1.0). Scales the applied phase:
 - $\alpha = 0$: No modulation – pure Gaussian output.
 - $0 < \alpha < 1$: Partial modulation – energy splits between central spot and rings.
 - $\alpha = 1$: Full modulation – all energy ideally goes into rings.
- **Quantization levels (Q):** Number of discrete phase levels (2, 4, 8, 16, 0 for continuous phase). Default is 0 (continuous phase). The beam structure depends strongly on Q :
 - $Q = 0$ (continuous phase): Ideal Bessel phase – a single ring with J_0^2 profile (for $\alpha = 1$).

- $Q = 8$: Close approximation to the ideal; most energy in the first ring, faint higher orders.
 - $Q = 4$: Moderate approximation; the first ring is dominant but may have slightly broadened shape.
 - $Q = 2$: Binary phase mask – produces multiple rings (odd orders) and each ring may exhibit a double-peak structure due to the abrupt phase jumps and the nature of the Hankel transform. No central spot appears for $\alpha = 1$ because the average transmission is zero.
- **Sampling accuracy (S)**: Multiplicative factor that increases the number of samples per 2π phase period beyond the base value (default: 1.0). The base number of samples per 2π is automatically set to $\max(16, Q)$ to ensure at least one sample per quantization level. The actual number of samples per period is then $S \times \max(16, Q)$. Increase S if you observe numerical artifacts or if the ring structure appears distorted.
 - **Export Designed Phase**: When enabled, system simulation pauses at the shaper plane and an export dialogue opens and allows for the export of the designed phase.

Simulation Model

The shaper applies a quantized radial phase mask, scaled by α . Within the paraxial Fourier optics framework, the field in the focal plane is the Fourier transform of the shaped field.

Ideal Radial Phase

The ideal continuous phase for a Bessel beam is linear in the radial coordinate:

$$\phi_{\text{ideal}}(\rho) = k_{\rho}\rho, \quad (1)$$

where $\rho = \sqrt{x^2 + y^2}$ is the radial coordinate in the shaper plane. In practice, the phase is taken modulo 2π because only the value modulo 2π affects the field.

From Fourier optics, the transverse wavenumber k_{ρ} determines the first ring radius in the focal plane:

$$r_0 = \frac{\lambda f k_{\rho}}{2\pi} \quad (2)$$

The focal spot size for a collimated Gaussian beam with radius w_0 is:

$$\Delta r_{\text{focus}} = \frac{\lambda f}{\pi w_0} \quad (3)$$

The ring separation ratio $M = r_0/\Delta r_{\text{focus}}$ yields the simple design equation:

$$\boxed{k_{\rho} = \frac{2M}{w_0}} \quad (4)$$

Quantized Radial Phase with Scaling

For a given quantization level $Q > 0$, the ideal phase modulo 2π is first quantized to the nearest of Q equally spaced levels, then scaled by α . The scaled quantized phase is:

$$\Phi_{\text{Bessel}}(\rho) = \alpha \cdot \frac{2\pi}{Q} \cdot \text{round} \left(\frac{(k_{\rho}\rho \bmod 2\pi)}{2\pi/Q} \right), \quad (5)$$

where round rounds to the nearest integer. For the continuous case ($Q = 0$), we simply take $\Phi_{\text{Bessel}}(\rho) = \alpha \cdot (k_\rho \rho \bmod 2\pi)$.

The complex transmission of the mask is $t(\rho) = \exp(i\Phi_{\text{Bessel}}(\rho))$.

Beam Characteristics for Different α and Q

The Fourier transform (Hankel transform) of the scaled phase mask yields an output that depends on both α and Q :

- For $\alpha = 0$, the mask has no phase modulation, so the output is simply the focused Gaussian beam (central spot only).
- For $0 < \alpha < 1$, the output contains both a central Gaussian component and ring structures. The central spot intensity decreases as α increases, while the ring energy grows.
- For $\alpha = 1$ and $Q = 0$ (continuous), the output is a single Bessel ring with intensity proportional to $J_0^2(k_\rho r)$.
- For $\alpha = 1$ and $Q = 2$ (binary), the mask acts as a radial square wave, producing multiple odd orders; each order may exhibit double-peak structures.
- For $\alpha = 1$ and $Q \geq 4$, the output progressively approaches the ideal single ring, with higher Q giving cleaner results.

Key Physical Principles

- **Fourier optics foundation:** With collimated input and shaper at distance f before the lens, the focal plane field is the exact Fourier transform with no quadratic phase distortion—essential for extended depth of focus.
- **Parameter α :** Controls the energy split between central spot and rings; a simple and intuitive power control.
- **Parameter Q :** Determines how faithfully the output approximates an ideal Bessel beam. Higher Q reduces artifacts (multiple orders, double peaks) and concentrates energy into the first ring.

Typical Application Scenarios

1. **Tunable laser processing:** Adjust α to control how much energy remains in the central spot vs. rings for applications requiring variable spot-to-ring energy ratios in drilling or cutting.
2. **Optical trapping with variable confinement:** Use α to tune between tight central trapping (α near 0) and extended ring trapping (α near 1) for multi-particle manipulation.
3. **Microscopy with adjustable background:** Control ring brightness to optimize the balance between central resolution and background illumination in imaging systems.
4. **Material processing parameter optimization:** Systematically explore α values to find the optimal energy distribution for specific material interactions.
5. **Beam shaping research:** Study how energy distribution between Gaussian and Bessel components affects propagation and self-reconstruction properties.
6. **Educational demonstrations:** Visually demonstrate the transition from pure Gaussian to pure Bessel-Gauss beams through continuous α variation.

Software Usage

This twin is available in the Digital Twin Hub. To achieve the optimal configuration for extended depth of focus, follow these steps:

System Setup

1. **Generate a Gaussian beam:** Place a Gaussian Beam Mode twin (SF-GAUS01) in your system.
2. **Collimate the beam (if needed):** If your source is not already collimated, add a collimation element:
 - Use an Ideal Lens [Collimation] twin (CF-ILC001) for perfect aberration-free collimation, OR
 - Use a Spherical Lens twin (CS-SLEN01) with appropriate curvature to achieve collimation

The beam at the shaper plane must have infinite wavefront curvature ($R_{\text{in}} \rightarrow \infty$).

3. **Add the beam shaper:** Place the Gauss-Bessel Beam Shaper [Power Control] twin (CF-BESP01) at a distance exactly equal to the focal length f before the Fourier lens.
4. **Configure the shaper:** Set the design parameters (M , α , Q , Sampling accuracy S).
5. **Add the Fourier lens:** Place a lens with focal length f in a 2-f configuration:
 - Use an Ideal Lens [2f-Setup] twin (CF-ILSU02) for perfect aberration-free Fourier transformation, OR
 - Use a Thin Lens twin (CF-THLE01) with focal length f placed at a distance f from the element plane.
6. **Observe the result:** Place field monitors (DF-FMON01) in the focal plane and beyond to observe the extended depth of focus of the ring-shaped Bessel-like beam.

Note:

- Input beam must be collimated at the shaper plane.
- Shaper must be positioned at distance f before the Fourier lens.
- For proper operation, the input field must be a Gaussian beam; other input fields produce unspecified outputs.

Exporting the designed phase for fabrication:

- Check the **Export Designed Phase** option in the shaper's dialogue.
- When enabled, system simulation pauses at the shaper plane and an export dialogue opens.
- The dialogue displays the current pixel size $\Delta x = \pi w_0 / (MN)$ where $N = S \cdot \max(16, Q)$. The user can specify the desired number of sampling points (i.e., the grid size) for the exported mask. The minimum allowed number of points corresponds to the current N (i.e., the sampling that would be used internally if the simulation continued). Larger grid sizes increase resolution but also file size.

- Users can adjust the number of points based on fabrication constraints; the software automatically resamples the phase data accordingly. However, reducing the number of points below the minimum would degrade the mask and is not permitted.
- After closing the export dialogue, the simulation continues normally.

⚠ Note: The Export Designed Phase option must be disabled during parameter runs or parametric optimizations, as it pauses the simulation and opens an export dialogue, which interrupts automated sweeps.

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Related Twins: SF-GBES01, CF-BESV01, CF-ILSU02